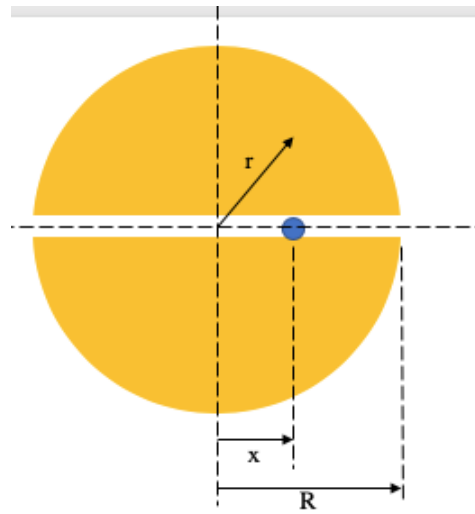
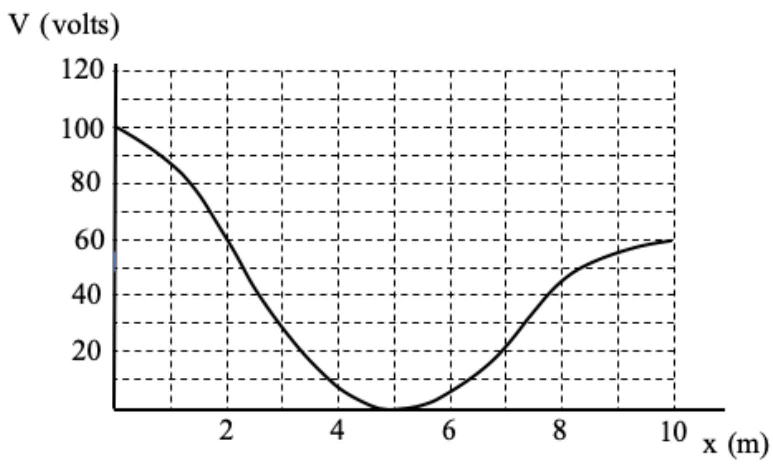
An insulating sphere of radius R = 10 m carries a volume charge density that changes both in sign and density as a function of r. A hole extends through the insulator. Inside the hole, located at x, is a non-conducting ball that carries a net positive charge. Both gravity and friction are negligible, and the ball does not exchange or accept charge from the sphere. The graph shows the electric potential V(r) for the region inside the hole due to the sphere.





a.) How far out from the center can the ball be released from rest and still exit the hole? Justify your response.

--this is a "potential well" problem—specifically;
--voltage is a modified potential energy function (V = U/q);
--that means that for an object to exit the ball at x = 10, it needs to have potential energy associated with a voltage of 60 volts.
--that means the ball *has* to start out with at the 60 volt mark if conservation of energy is to hold;
--and that means the ball has to begin anywhere from z = 0 to x = 2 meters . . . with x = 2 meters being the farthest out.

b.) The ball is released from rest from the position identified in Part a, and exits the hole. Describe the magnitude and direction of its acceleration between r = 5 m and r = 10 m. Explain your reasoning.

--at x = 0, the voltage (hence potential energy) is zero and all of the body's energy is wrapped up in kinetic energy;
--as it proceeds to the right, it slows losing kinetic energy and gaining kinetic energy;
--this suggests the acceleration is opposite the direction of motion, or to the left;
--as for magnitude, the electric field (which is a modified force field) is equal to the -dV/dx, so the field and hence the force on the charge will be largest where the slope is greatest;
--that occurs between around x = 5 and x = 8;
--between x = 8 and x = 10 meters, the acceleration tails off.

c.) It is suggested that the expression E(r) = A(B - r) models the electric field in the hole, where $0 < r < 10$ m. Assuming rightward is positive, A = 4 V/m^2 and B = 5 m, state one feature of this model that does NOT correctly model the electric field in this situation. Justify your response.

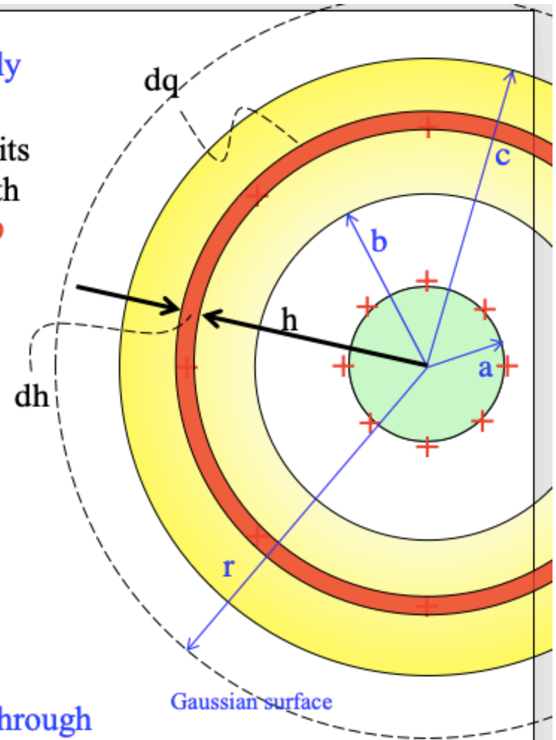
--at r = 0, E(r) is positive, so that checks out;
--at r = 4.5 meters, E(r) is still positive, so that's OK;
--at r = 5 meters, E(r) is zero; this is correct as E(r) = - dV/dr, and the slope is zero.
--the function suggests symmetry before and after x = 5, which isn't the case;
--also, the function has zero slope at x = 10, so E should be zero at x = 10 . . . but the function maintains E = 4 (5 – 10) = -20, which isn't zero.

d.) Explain the steps one would have to execute to derive an expression for $\rho(r)$ using $E(r)$.

--according to Gauss's Law, the electric flux through an arbitrary Gaussian sphere centered on the charged sphere must be proportional to the charge enclosed divided by epsilon-not.
--the charge enclosed inside the Gaussian sphere of radius "r" would require the creation of a differentially thin spherical shell of radius "c" (a dumby variable) and volume dV, where dV was the area of the sphere's surface 4(pi)c^2 times the differential thickness dr, and the charge enclosed inside the sphere was dq = p(r) dV;
--analyze the left side of Gauss's Law . . . E (4(pi)r^2) and putting it equal to the integral evaluated between r = 0 and r = R of p(r)dV divided by epsilon-not,
--then substituted in the given function for E(r) and solve for the only unknown in the equation, p(r).

Note that the mathematics of this is shown on Slide 24 of the Gauss's Law PowerPoints. I've provided a picture of the first slide (the one that defines rho) on the next page:

Define a differential volume *dV* as a differentially *thin shell* of radius *h* and thickness *dh.* Its magnitude will be its surface area $(4\pi h^2)$ times its differential thickness *dh*, or $dV = (4\pi h^2 dh)$. With a given volume charge density of $\rho = kh$, where ρ has been evaluated at *h*, we can write:



$$\rho = \frac{dq}{dV}$$

$$\Rightarrow \quad dq = \rho dV$$

$$= \rho(4\pi h^2 dh)$$

$$= (kh)(4\pi h^2 dh)$$

$$= 4\pi k h^3 dh$$

With this, we can determine the charge shot through the entire shell, or through just part of the shell.

24.)